# Grade 9/10 Math Circles

## April 3, 2024

## Probability III - Problem Set

## In-Lesson Exercises

1. How does the answer to example 1 change if the PIN is 6 digits long?
2. Suppose you flip a coin 3 times. What is the probability that you flip heads exactly two times, given that you flip heads at least once?
3. Men are surprisingly likely to be colour-blind. About 1/12 of men are colour-blind while only 1/200 of other people are colour-blind. Suppose that men make up 50% of the population.
   (a) What is the probability that a random person is colour-blind?
   (b) Given that someone is colour-blind, what is the probability that they are a man?
4. Continuing with the setup from the false negative problem, what is the probability that you are healthy, given that your test result was positive?
5. Determine if each of the following are a random variable:
   (a) The number of face cards in a hand of 4 cards.
   (b) The winner of a chess tournament.
   (c) The number of games a player in a chess tournament wins.
6. Find the expected number of heads if you flip 2 coins.
7. Your friend suggests that you bet on the outcome of a die roll. If the roll is 1 through 4, they will pay you \$3. If the roll is 5 or 6, you will pay them \$$k$. If $k = 7$, should you play the game?

   Bonus: What value of $k$ would make the game fair? (Neither you nor your friend expect to make money.)